Mathematical model on thin layer drying of olive fruit (*Olea europeae* L.)

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Abstract The thin layer drying characteristics of olive (Variety Zarrazi) was investigated under three microwave power densities: 0.5, 1 and 2 W/g. Drying time decreased considerably with increase in microwave power density. Analysis of the drying curves shows that there is no constant rate period, but that the period of gradual reducing of the rate was present. The drying data were fitted to Lewis, Henderson and Pabis, Two term model, Wang and Singh, Parabolic, Logistic, Modified Page, Page, and Two-term exponential equations during a time interval equal to 30 min. The Two-term exponential ($R^2 = 0.99941; \chi^2 = 5.7\times10^{-6}$ and RMSE= 0.00195) gave the best fit to predict the microwave drying of olive. The values of the effective diffusivity coefficients of olive varied between $2.54\times10^{-9}$ and $12.60\times10^{-9}$ m$^2$/s while the activation energy was 1.089 W/g. The lower activation energy indicates that drying of olive requires less energy and is hence a cost and energy-saving method.

Keywords: olive, microwave drying, Effective diffusivity, activation energy, modeling.

Introduction

The evergreen olive cultivar (*Olea europea* L., Oleaceae) is an important Mediterranean tree (Kiple and Ornelas, 2000). Tunisian oleiculture constitutes one of the principal economical and agricultural strategic sectors that are known for their richness of varieties (Abaza et al., 2001). Now, the olive plantation, occupying about 1.6 million ha, is dominated by three main cultivars (Issaoui et al., 2008). The Chétouï is omnipresent in the north, the Zarrazi is omnipresent in the south, while the Chemlali is ubiquitous to the rest of the country (Grati-Kamoun et al., 2006).

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The olive-growing areas spread from the northern to the southern regions, where a wide range of edaphic-climatic conditions are prevailing. These three varieties account for 97% of the total olive tree orchards and contribute more than 92% of the national production of olive oil.

Virgin olive oil is a valuable vegetable oil extracted from fresh and healthy olive fruits (Olea europeae L.) by mechanical processes and without any preliminary refining (Garcia and Yousfi, 2006). It has an excellent nutritional, functional and sensorial qualities (Matos et al., 2007), and is a product of major economical importance in Tunisia. Regarding the wholesome advantages of olive oil consumption, it owns a high antioxidant capability and reduces the risk of suffering from cardiovascular diseases and contracting breast or colon cancers (Cicerale et al., 2010; Gigon and Le Jeune, 2010).

Healthy eating was one of the most important factors in food choice among Tunisian citizens. They were conscious that more frequent consumption of fruit and vegetables should be a part of a healthy diet. One of the most important of these fruits and vegetables is the olive fruit. The olive fruit has an excellent nutritional, functional and sensorial qualities, and it owns a high antioxidant capability. Fruits and vegetables are often dried by sunlight or hot air. Sun and hot air dryings of white mulberry were investigated (Doymaz, 2004; Akpınar, 2008). However, there are many problems in sun drying such as the slowness of the process, the exposure to environmental contamination, uncertainty of the weather, and the manual labor requirement (Maskan and Gogus, 1998). On the other hand low-energy efficiency and lengthy time during the falling rate period are major disadvantages of hot air drying of foods, because heat transfer to the inner sections of foods during conventional heating is limited by the low thermal conductivity of food materials in this period (Maskan, 2000). Due to these difficulties, more rapid, safe and controllable drying methods are required. Also, it is necessary to dry the product with minimum cost, energy and time. In microwave drying, drying time is shortened due to quick absorption of energy by water molecules, causes rapid evaporation of water, resulting in high drying rates of the food (Soysal et al., 2006; Darvishi et al., 2013).

One of the most important aspects of drying technology is the modeling of the drying process. The present research is focused on this issue. Therefore, the aim of this study was to (i) evaluate a suitable drying model for describing the microwave drying process of olive, (ii) study the effect of power density on the drying kinetics of olive, (iii) calculate the effective moisture diffusivity and activation energy.
Materials and methods

Sample preparation

To determine the initial moisture content, three 10 g of samples was dried in an oven (Memmert UM-400) at 105 ºC for 24 h. The initial moisture content of olive was calculated 46 ± 2% (% d.b.) as an average of the results obtained.

Experimental equipment and procedure

Drying tests were carried out in the microwave oven (Bosch, type HMT84M651). It is a digital furnace domesticates, its following design features: The microwaves are emitted there at a frequency of 2450 MHz; it makes it possible to operate on 5 different levels of power, namely, 90 W, 180 W, 360 W, 600 W and 900 W. Its room for drying measures 215 x 337 x 354 mm of volume; it has a plate out of glass 315 mm in diameter which can carry out 5 turns per minute and whose direction of rotation to 360° can be reversed while pressing on the button" On/Stop". The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. Experiments were performed at three initial masses of 45, 90 and 180 g at a microwave power of 90 W (or power densities (microwave power/mass) of 2, 1, 0.5 W/g). The moisture losses of the samples were recorded at 5 min intervals during the drying process by an analytical balance (Sartorius, Model CP2245) with a precision of ± 0.01 g. After the set time, the sample was taken out of the drying chamber, weighed on the analytical balance (accuracy of 0.01 g) and placed back into the chamber within 10 s (Karaaslan and Tuncer, 2008).

Modeling of drying process

The moisture ratio of olive samples during the thin layer drying experiments was calculated using the following equation:

\[ MR = \frac{X - X_e}{X_0 - X_e} \]  

(1)

Where \( X \) is the moisture content at any time \( t \) (kg/kg dry solid); \( X_0 \) is the initial moisture content (kg/kg dry solid), and \( X_e \) is the equilibrium moisture content (kg/kg dry solid). The values of \( X_e \) are relatively small compared to \( X \) and \( X_0 \), hence the error involved in the simplification by assuming that \( X_e \) is equal to zero is negligible (Aghbashlo et al., 2008), thus moisture ratio was calculated as:

\[ MR = \frac{X}{X_0} \]  

(2)
For drying model selection, drying curves were fitted to 9 well-known thin layer drying models which are given in Table 1. The best fit was determined using three parameters: higher values for the coefficient of determination ($R^2$), the reduced chi-square ($\chi^2$) and root mean square error (RMSE) using Equations (3-5), respectively (Yaldiz and Ertekin, 2004; Gündahan et al., 2005; Özdemir and Devres, 1999). The statistical analyses were carried out using SPSS 15 software. Where $MR_{pre,i}$ is the $i$th predicted moisture ratio, $MR_{exp,i}$ is the $i$th experimental moisture ratio, $N$ is the number of observations and $z$ is the number of constants in the drying model (Akpinar, 2006).

$$R^2 = \frac{\sum_{i=1}^{N}(MR_{pre,i} - MR_{exp,i})^2}{\sum_{i=1}^{N}(MR_{pre,i} - \bar{MR}_{exp})^2}$$ (3)

$$\chi^2 = \frac{\sum_{i=1}^{N}(MR_{pre,i} - MR_{exp,i})^2}{N - z}$$ (4)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(MR_{pre,i} - MR_{exp,i})^2}{N}}$$ (5)

**Table 1. Thin-layer drying models**

<table>
<thead>
<tr>
<th>No.</th>
<th>Model name</th>
<th>Model equation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lewis</td>
<td>$MR = \exp(-kt)$</td>
<td>Lewis, 1921</td>
</tr>
<tr>
<td>2</td>
<td>Henderson &amp; Pabis</td>
<td>$MR = ae^{x(-kt)}$</td>
<td>Henderson and Pabis, 1961</td>
</tr>
<tr>
<td>3</td>
<td>Two term model</td>
<td>$MR = ae^{x(-k_0t)} + be^{x(-k_1t)}$</td>
<td>Henderson, 1974</td>
</tr>
<tr>
<td>5</td>
<td>Parabolic</td>
<td>$MR = at^2 + bt + c$</td>
<td>Sharma and Prasad, 2001</td>
</tr>
<tr>
<td>6</td>
<td>Logistic</td>
<td>$MR = a_0/(1 + ae^{x(kt)})$</td>
<td>Chandra and Singh, 1995</td>
</tr>
<tr>
<td>7</td>
<td>Modified Page</td>
<td>$MR = ae^{x(-(kt)^n)}$</td>
<td>White et al., 1981</td>
</tr>
<tr>
<td>8</td>
<td>Page</td>
<td>$MR = ae^{x(-(kt)^n)}$</td>
<td>Page, 1949</td>
</tr>
<tr>
<td>9</td>
<td>Two-term exponential</td>
<td>$MR = ae^{x(-(kt)^n)} + (1 - a)e^{x(-(kat)}}$</td>
<td>Sharaf-Elden et al., 1980</td>
</tr>
</tbody>
</table>
Evaluation of effective diffusivities

The drying characteristics of the falling rate period can be described by using Fick’s diffusion equation (Crank, 1975), Eq. (6) can be used to evaluate effective diffusivity of spherical particles

\[
\frac{X}{X_0} = \frac{6}{\pi^2} \sum_{n=0}^{\infty} \left( \frac{1}{n^2} \exp \left(-\frac{D_{\text{eff}}n^2\pi^2}{R^2} \right) \right) t \quad (6)
\]

where, \(D_{\text{eff}}\) is the effective diffusivity and \(R\) is the radius of the grain. For long drying times Eq. (6) can be further simplified to the first term of the series (Tutuncu and Labuza, 1996). Thus Eq. (3) can be written in the logarithmic form as Eq. (7):

\[
\ln(MR) = \ln \frac{6}{\pi^2} - \left(\frac{D_{\text{eff}}\pi^2}{R^2} \right) t \quad (7)
\]

Diffusion coefficients were determined by plotting \(\ln MR\) verses drying time, \(t\). The plot yields a straight line with the slope of \(\pi^2D_{\text{eff}}/R^2\) from which the effective diffusivity was evaluated.

Power density dependence of diffusivity and calculation of activation energy

Inasmuch as the temperature is not precisely measurable inside the microwave dryer, the activation energy is found as modified from the revised Arrhenius equation. In this method it is assumed as related to the effective diffusion coefficient and the ratio of microwave output power to sample weight (\(P_d\)) instead of to air temperature. Then Equation (8) can be effectively used (Dadali et al., 2007) as follows:

\[
D_{\text{eff}} = D_0\exp \left(-\frac{E_a}{P_d} \right) \quad (8)
\]

where \(E_a\) is the activation energy (W/g), \(m\) is the mass of raw sample (g), and \(D_0\) is the pre-exponential factor (m²/s).

Results and discussion

Experimental drying curves

The influence of microwave power and drying air temperature on the moisture ratio versus drying time curve is shown in Figure 1. It is apparent that the moisture ratio decreases continuously with drying time. As can be seen from the data presented, the time required to dry olive to \(Y\%\) (% d.b.) moisture
content decreased with an increase in microwave power density from 0.5 to 2 W/g. Of the 12% (% d.b.) moisture content the drying time varied between 30 and 55 min (Table 2) as the microwave power density increased from 0.5 to 2 W/g.

![Drying curves of moisture ratio with drying time at different microwave power densities.](image)

**Fig. 1.** Drying curves of moisture ratio with drying time at different microwave power densities.

The time was reduced from 55 to 40 min (Table 2) during drying of olive at 1 W/g microwave power densities. Increasing the microwave density to 2 W/g further decreased the required drying time to 30 min. In general, increasing the microwave power density by 0.5 and 1 W/g starting from 0.5 to 1, or 2 W/g reduced the drying time by 27%, 46%, respectively (Table 2), compared to the drying time required to reduce moisture content to 12% (% d.b.) at 0.5 W/g drying power density.

Analysis of the drying curves shows that there is no constant rate period, but that the period of gradual reducing of the rate was present (Figure 2). A rapid drying was observed in the moisture ratio in this study which is in agreement with previous findings (Massamba et al., 2012; Minaei et al., 2011; Ekow et al., 2013). The instantaneous moisture ratio rapidly decreases as the microwave power density increases which is due to faster moisture diffusion from the centre of olive. This phenomenon indicated that the mass transfer of drying sample was rapid during microwave heating because the microwave penetrated directly into the sample. The heat generated in inside the sample and provided fast and uniform heating throughout the entire product, thus creating a large vapour pressure differential between the centre and the surface of the product and allowing rapid transport and evaporation of water. An increase in microwave power density significantly shortened the drying time.
Fig. 2. Variations of drying rate at different microwave power densities.

Table 2. Effects of different drying power densities on drying time of olive.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Power density (w/g)</th>
<th>Time (min)</th>
<th>% reduction in drying time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>55</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>40</td>
<td>27%</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>30</td>
<td>46%</td>
</tr>
</tbody>
</table>

Evaluation of the models

The moisture ratio calculated from the drying data at different power densities was fitted to the thin layer models given in Table 1. The statistical regression results of different models, including the drying model coefficients were listed in Table 3.

Table 3. Statistical results of different thin layer models

<table>
<thead>
<tr>
<th>N°. Model name</th>
<th>( P_d )</th>
<th>Model constants</th>
<th>( R^2 )</th>
<th>( \chi^2 )</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Lewis</td>
<td>0.5</td>
<td>( k = 0.00728 )</td>
<td>0.88967</td>
<td>0.000990</td>
<td>0.02739</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>( k = 0.0199 )</td>
<td>0.88784</td>
<td>0.00485</td>
<td>0.06357</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( k = 0.02203 )</td>
<td>0.90026</td>
<td>0.00807</td>
<td>0.08201</td>
</tr>
<tr>
<td>2. Henderson &amp;</td>
<td>0.5</td>
<td>( a = 1.03211 )</td>
<td>0.93198</td>
<td>0.00066</td>
<td>0.02098</td>
</tr>
<tr>
<td>Pabis</td>
<td>1</td>
<td>( a = 1.07448 )</td>
<td>0.92653</td>
<td>0.00384</td>
<td>0.05060</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( a = 1.09169 )</td>
<td>0.92838</td>
<td>0.00694</td>
<td>0.06802</td>
</tr>
<tr>
<td>3. Two term model</td>
<td>0.5</td>
<td>( a = 0.53748 )</td>
<td>( k_0 = 0.00879 )</td>
<td>( k_1 = 0.51567 )</td>
<td>0.00879</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>( a = 0.54492 )</td>
<td>( k_0 = 0.03678 )</td>
<td>( k_1 = 0.54677 )</td>
<td>0.03678</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( a = -0.00022 )</td>
<td>( b = -0.00135 )</td>
<td></td>
<td>0.39745</td>
</tr>
</tbody>
</table>
For all models, the $R^2$, $\chi^2$, and RMSE were higher than 0.88874 and lower than 0.01156 and 0.08201 respectively. Two-term exponential model provided the highest $R^2$ and lowest $\chi^2$ and RMSE, thus, it was selected for predicting the moisture ratio of olive. Validation of the selected model was confirmed by comparing the predicted moisture ratio with the measured values at different power densities.

Figure 3 compares experimental data with those predicted with the Two-term exponential model for olive at 0.5, 1 and 2W/g. There was a very good agreement between the experimental and predicted moisture ratio values, which closely banded around a 45° straight line. Figure 4 present the variation of experimental and predicted moisture ratio using the best model with drying time for dried olive. Two-term exponential model gives a good estimation for the drying process.

The Two-term exponential model has also been suggested by others to describe the effect of microwave power, air velocity and temperature on the final drying of osmotically dehydrated bananas (Pereira et al., 2007), effects of drying methods on the composition of thyme (Calín-Sánchez et al., 2013) and Dehumidified Air, Heat Pump and Convective Cum Vacuum Microwave Drying Characteristics and Models (Law et al., 2010).
Fig. 3. Comparison of experimental and calculated moisture ratio values by the two-term exponential model.

Hence, the effect of power density on the drying constants of the Two-term exponential model was taken into account in developing the relation between these constants and the drying power density. The regression equations relating the constants of the selected model and the drying power density are the following:

\[ MR = a \exp(-kt) + (1 - a) \exp(-kat) \]

where \( a, k \) are constants.

\[ k = -0.00874 + 0.07297 \times P_d - 0.01826 \times P_d^2 \quad R^2 = 0.9999 \]
\[ a = 1.93073 + 0.25005 \times P_d - 0.07342 \times P_d^2 \quad R^2 = 0.9999 \]

Fig. 4. Drying curves for the experimental data and that predicted based on the two-term exponential model.
**Moisture diffusivity and activation energy**

To calculate the effective diffusivity by using the method of slopes, the logarithm of moisture ratio values, \( \ln (MR) \), were plotted against drying time (\( t \)) according to the experimental data obtained at various microwave output power density. The linearity of the relationship between \( \ln (MR) \) and drying time is illustrated in Figure 5. It was determined that, the effective diffusivity of olive varied from \( 2.54 \times 10^{-9} \) to \( 12.60 \times 10^{-9} \) \( \text{m}^2/\text{s} \) over the microwave power density range studied. Values of effective diffusivities of olive determined under the microwave power density range of 0.5, 1, 2 \( \text{W/g} \) are given in Table 4. As expected, the values of diffusivities increased with the increase of microwave power density due to the increase of temperature and consequently water vapor pressure. The increase in power resulted in rapid heating of the product, thus increasing the vapor pressure inside the product that made the diffusion of moisture towards the surface faster.

\[
\text{Fig. 5. lnMR vs drying time (s)}
\]
Fig. 6. Variation of effective diffusivity as function of power densities

Table 4. Effective diffusivities of olive at different microwave power densities

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Power density (w/g)</th>
<th>Diffusivity (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>2.54×10⁻⁹</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>7.20×10⁻⁹</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>12.60×10⁻⁹</td>
</tr>
</tbody>
</table>

The values of effective diffusivity versus \( P_d \) shown in Figure 6 and table 4 accurately fit to Eq. (10) with the coefficient of determination (\( R^2 \)) of 0.9999. Then, \( D_0 \) and \( E_a \) values were estimated as 2.17×10⁻⁸ m²/s and 1.089 W/g. The dependence of the effective diffusivity of olive samples on the power density can be represented by the following equation:

\[
D_{\text{eff}} = 2.17 \times 10^{-8} \exp \left( -\frac{1.089}{P_d} \right) \tag{10}
\]

Conclusion

Thin layer drying experiments was conducted to determine the thin layer drying characteristics and activation energy of olive in a microwave oven. Nine thin layer drying models were evaluated for their suitability. The Two-term exponential model given by \( \text{MR} = \text{ae}^{\text{xp}(-kt)} + (1 - \text{a})\text{exp}(-\text{kat}) \) represented the microwave drying characteristics of olive better than the other frequently used thin layer drying models proposed. The effective diffusivity varied from 2.54×10⁻⁹ to 12.60×10⁻⁹ m²/s, by increasing microwave power
density. An Arrhenius relation with an activation energy value of 1.089 W/g expressed the effect of power density on the diffusivity. This study indicates that Two-term exponential model gave an excellent fitting to the drying experimental data of olive and we obtain correlations afterwards:
\[ k = -0.00874 + 0.07297 \times P_d - 0.01826 \times P_d^2 \quad R^2 = 0.9999 \]
\[ a = 1.93073 + 0.25005 \times P_d - 0.07342 \times P_d^2 \quad R^2 = 0.9999 \]

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