
Mass and volume modelling of Persian lime (*Citrus aurantifolia*) with geometrical attributes

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A.N. Lorestani, F. Jaliliantabar and R. Gholami (2012) Mass and volume modelling of Persian lime (*Citrus aurantifolia*) with geometrical attributes. Journal of Agricultural Technology 8(5): 1537-1543.

Fruits are often graded on the basis of size and projected area, but it may be more suitable and/or economical to develop a machine which grades by mass and/or volume. Therefore, relationships between mass/volume and dimensions or projected areas of fruits are needed. This information would be used to design and develop sizing systems. Models for predicting mass and volume of lime from its dimensions and projected areas were identified. Models were divided into three classifications: 1-Single and multiple variable regressions of lime dimensions (1st classification), 2-Single and multiple variable regressions of projected areas (2nd classification), 3-Estimation of lime shape; ellipsoid or spheroid based on volume (3rd classification). The 3rd classification models gave the highest performance followed by 2nd and 1st classifications, respectively, which R^2 is closed to unity. Among single variable models, the volume model versus smallest diameter was power and gave maximum coefficient of determination, $R^2=0.87$.

Key words: physical properties, modelling, mass, volume, Persian lime, *Citrus aurantifolia*

Nomenclature: M= fruit mass, g; V= fruit Volume, cm³; GM = geometric mean diameter, mm; a= the longest intercept diameter of fruits, mm; b= the intermediate intercept diameter of fruits normal to a, mm; c= the minor intercept diameter of fruits normal to a and b, mm; PA = first projected area, mm²; PB = second projected area, mm²; PC = third projected area, mm²; k_i = Regression coefficients ; m= constant; SPH= sphericity(%); D= density, g.ml⁻¹; SD= standard deviation; CV= coefficient of variation is equal to ratio of SD per average(%).

Introduction

Persian Lime (*Citrus aurantifolia*) is a variety of lime. Dried Persian lime in Middle East, especially in Iran, is used as a flavoring in foods. This fruit is widely produced in Iran. This acid lime lacks the long history. Its identity has been in doubt and only in recent years has it been given the botanical name. An alternate common name is Tahiti lime or Bearss lime

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(http://simple.wikipedia.org/wiki/Persian_lime). Physical characteristics of agricultural products are the most important parameters in design of grading, conveying, processing and packaging systems. Among these physical characteristics, mass, volume and projected area are the most important ones in sizing systems. Other important parameters are width, length, and thickness (Lorestani and Tabatabaefar, 2006).

The objective of this research was to determine an optimum dried lime mass and volume models based on its dimensions. This information would be used to design and develop sizing systems.

Materials and methods

Dried Lime fruits used for all the experiments were purchased from the local market of Kermanshah, Iran for this study. 100 samples of lime fruit are prepared and kept in 25 °C in the laboratory.

The mass (M) of each lime fruit was measured to 0.01 g with accuracy on a digital balance. Its volume (V) was obtained by volume of water displaced. A lime fruit was submerged into a known volume of water and then the volume of water displaced was measured (Akar and Aydin, 2005). Water temperature was kept at 25°C.

Density (D) of each lime fruit was calculated by the mass of lime fruit in air divided by its volume. Three mutually perpendicular axes: a represented the longest intercept, b represented the longest intercept normal to a, and c represented the longest intercept normal to a and b, of lime fruit were measured to 0.01 mm by a caliper (Tabatabaefar *et al.*, 2000; Lorestani and Tabatabaefar, 2006). Three mutually perpendicular areas, PA, PB, and PC were measured by positioning each lime fruit in the diameter directions (Ndukwu, 2009). Geometric mean diameter (GM), was determined from the cubic roots of the three diameters, $(abc)^{1/3}$, and percent sphericity (SPH) was equal to the geometric mean diameter (GM) divided by the longest diameter, $(GM/a) \times 100$ (Mohsenin, 1986). These properties are given in Table 1.

Regression models (linear, non-linear, single and multiple variables) Spreadsheet software, Microsoft Excel 2010 and SPSS 17.0 Software were used to analyze the data and to determine regression models between the parameters of either linear or polynomial form.

In order to estimate a lime fruit's mass and volume from measured dimensions (length and projected area), the following three categories of models were suggested (Tabatabaefar *et al.*, 2000; Ghabel *et al.*, 2010).

Regression models of mass with major (a), intermediate (b), minor (c) and all three diameters were applied (Lorestani and Tabatabaefar, 2006; Mirzaee *et al.*, 2008). Regression models of mass with each projected area (PA,

PB, and PC) and all three projected areas were determined. Regression models of mass with Lime measured volume (V). Three models (Tables 2, 3, 4) were determined. In total 9 models for all three categories were determined.

For first category, the independent variables were one or three mutually perpendicular diameters.

$$\begin{aligned} M &= k_1a + k_2b + k_3c + k_4 \\ V &= k_1a + k_2b + k_3c + k_4 \end{aligned} \quad (1)$$

Where: M- mass of the Lime (g); a, b, c- the longest, median and the smallest diameters respectively (mm); k_i - regression coefficients. In this category, the mass can be estimated as a function of one, two or three diameters.

For the second category, the independent variables were three mutually perpendicular projected areas.

$$\begin{aligned} M &= k_1PA + k_2PB + k_3PC + k_4 \\ V &= k_1PA + k_2PB + k_3PC + k_4 \end{aligned} \quad (2)$$

Where: PA, PB, PC- projected areas in a diameter directions (mm^2).

For the third category, the mass can be estimated as a function of the volume:

$$M = k_1V_{\text{psp}} + k_2 \quad (3)$$

$$M = k_1V_{\text{ell}} + k_2 \quad (4)$$

$$M = k_1V + k_2 \quad (5)$$

Where:

$$V_{\text{PSP}}\text{- volume of prolate spheroid (mm}^3\text{)} = \frac{4\pi}{3} (a/2)(b/2)^2,$$

$$V_{\text{ell}}\text{- volume of ellipsoid (mm}^3\text{)} = \frac{4\pi}{3} (a/2)(b/2)(c/2).$$

Results and discussions

The physical attributes of Persian variety of Lime such as major, minor, and intermediate diameter, mass, volume, density, geometric mean diameter, and percent sphericity, of Limes are shown in Table 1.

Table 1. Physical attributes of dried Persian Lime fruit

Statistical values	a (mm)	b (mm)	c (mm)	M (g)	V (ml)	D (g.ml ⁻¹)	GM (mm)	SPH (%)	PA (mm ²)	PB (mm ²)	PC (mm ²)
Min.	25.83	25.02	24.96	2.06	8.65	0.17	25.27	83.91	532.59	597.75	612.50
Max.	43.24	38.42	38.22	7.25	31.60	0.35	38.94	99.7	1423.22	1625.90	1637.90
Ave.	35.19	32.97	32.99	4.87	20.59	0.24	33.68	96.02	1052.87	1173.84	1181.58
SD	3.63	2.62	2.56	1.03	4.76	0.26	2.73	4.65	182.19	200.50	206.12
CV (%)	10.31	7.93	7.77	21.17	23.11	10.89	8.02	4.85	17.30	17.08	17.44

a, b, c - diameters; M - mass; V - volume; D - density; GM - geometric mean diameter; SPH - sphericity; PA, PB, PC- projected areas

A total of 9 regression models in three different categories have been classified which Shahi *et al.* (2009) who also reported. Coefficient of determination (R^2) and models that obtained from the data for Iranian variety of Limes are shown in Table 2. All of the models coefficients were analyzed with F-test and T-test by SPSS Software, all of them were significant at $\alpha=5\%$. Among the first category models, model number 4 had the higher R^2 while, for this model, measurement of three diameters is needed. So among the models (nos. 1, 2, 3), model number 3 for total of observations, had higher R^2 than the other models. Therefore, model number 3 obtained which based on the smallest diameter (c). Thus, sizing of Limes based on the smallest diameter is suitable as reported by Lorestani and Tabatabaeefar (2006) and Lorestani *et al.* (2011).

Among the linear regression projected area models (nos. 5, 6, 7, 8), model number 8 for *Citrus aurantifolia* variety of Limes had higher R^2 than the other models. This model requires measurement of three projected areas, so it is not economical. Therefore, other models (nos. 5, 6, 7), model number 7 had higher R^2 . These results were in agreement with previous works by other authors (Tabatabaeefar *et al.*, 2000; Tabatabaeefar and Rajabipoor, 2005; Lorestani and Tabatabaeefar, 2006; Mirzaee *et al.*, 2008; Lorestani *et al.*, 2011; Lorestani and Ghari, 2012).

Among the linear regression based on volume (nos. 9, 10, 11), model number 11 is based on volume of ellipsoid, had higher R^2 . Therefore, this model for sizing of Limes is recommended. These results were in agreement with previous works by other authors (Tabatabaeefar *et al.*, 2000; Tabatabaeefar and Rajabipoor, 2005; Lorestani and Tabatabaeefar, 2006; Mirzaee *et al.*, 2008; Lorestani *et al.*, 2011).

In order to consider the models for the total of observations, similar models were obtained, that are shown in Table 2. Nonlinear regression models (polynomial and power) are also shown in Tables 3 and 4, respectively. These models were used only for comparison with linear regression models. We

concluded that the linear regression models gave higher R^2 than the other models, and economical models for application.

Table 2. Coefficient of determination (R^2) and linear regression models

No.	Models (MB)	R^2	Models (VB)	R^2
1	$M = k_1a + k_2$	0.60	$V = k_1a + k_2$	0.63
2	$M = k_1b + k_2$	0.77	$V = k_1b + k_2$	0.84
3	$M = k_1c + k_2$	0.80	$V = k_1c + k_2$	0.85
4	$M = k_1a + k_2b + k_3c + k_4$	0.84	$V = k_1a + k_2b + k_3c + k_4$	0.90
5	$M = k_1PA + k_2$	0.81	$V = k_1PA + k_2$	0.84
6	$M = k_1PB + k_2$	0.81	$V = k_1PB + k_2$	0.89
7	$M = k_1PC + k_2$	0.82	$V = k_1PC + k_2$	0.90
8	$M = k_1PA + k_2PB + k_3PC + k_4$	0.86	$V = k_1PA + k_2PB + k_3PC + k_4$	0.92
9	$M = k_1V + k_2$	0.79	$V = k_1M + k_2$	0.79
10	$M = k_1V_{psp} + k_2$	0.82	-	-
11	$M = k_1V_{ell} + k_2$	0.84	-	-

VB: volume based, MB: Mass based, k_i : regression coefficients

Table 3. Coefficient of determination (R^2) and polynomial regression models

No.	Models (MB)	R^2	Models (VB)	R^2
1	$M = k_1a^2 + k_2a + k_3$	0.61	$V = k_1a^2 + k_2a + k_3$	0.62
2	$M = k_1b^2 + k_2b + k_3$	0.77	$V = k_1b^2 + k_2b + k_3$	0.85
3	$M = k_1c^2 + k_2c + k_3$	0.81	$V = k_1c^2 + k_2c + k_3$	0.86
4	$M = k_1PA^2 + k_2PA + k_3$	0.81	$V = k_1PA^2 + k_2PA + k_3$	0.84
5	$M = k_1PB^2 + k_2PB + k_3$	0.81	$V = k_1PB^2 + k_2PB + k_3$	0.89
6	$M = k_1PC^2 + k_2PC + k_3$	0.83	$V = k_1PC^2 + k_2PC + k_3$	0.90
7	$M = k_1V^2 + k_2V + k_3$	0.79	$V = k_1M^2 + k_2M + k_3$	0.79
8	$M = k_1V_{psp}^2 + k_2V_{psp} + k_3$	0.82	-	-
9	$M = k_1V_{ell}^2 + k_2V_{ell} + k_3$	0.84	-	-

VB: volume based, MB: Mass based, k_i : regression coefficients

Table 4. Coefficient of determination (R^2) and power regression models

No.	Models (MB)	R^2	Models (VB)	R^2
1	$M = k_1 a^m$	0.62	$V = k_1 a^m$	0.64
2	$M = k_1 b^m$	0.78	$V = k_1 b^m$	0.85
3	$M = k_1 c^m$	0.82	$V = k_1 c^m$	0.87
4	$M = k_1 PA^m$	0.81	$V = k_1 PA^m$	0.85
5	$M = k_1 PB^m$	0.82	$V = k_1 PB^m$	0.88
6	$M = k_1 PC^m$	0.83	$V = k_1 PC^m$	0.90
7	$M = k_1 V^m$	0.80	$V = k_1 M^m$	0.80
8	$M = k_1 V_{psp}^m$	0.83	-	-
9	$M = k_1 V_{ell}^m$	0.85	-	-

VB: volume based, MB: Mass based, k_i : regression coefficients, m: constant

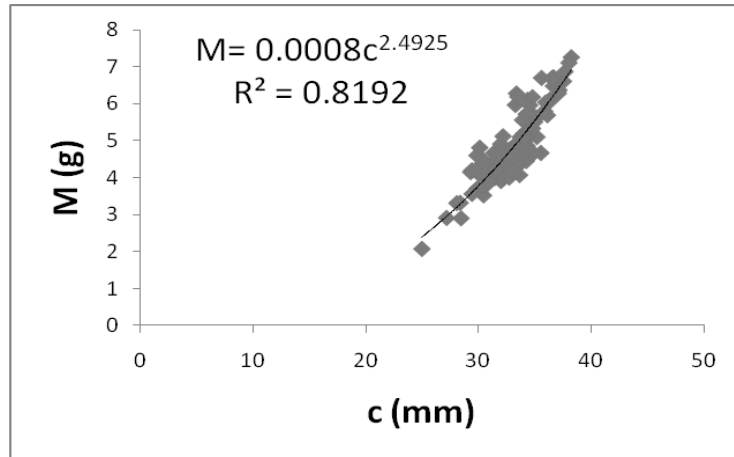


Fig. 1. lime mass model based on smallest diameter

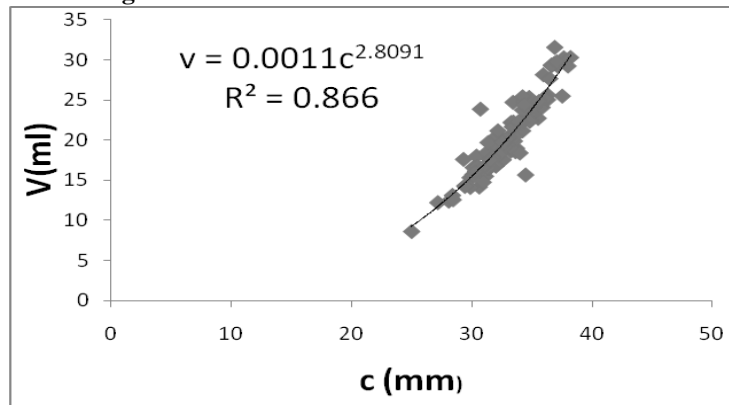


Fig. 2. lime volume model based on smallest diameter

Conclusion

Mostly, modelling based on volume of lime has a higher coefficient of determination in comparison with mass modelling.

The recommended equation to calculate Lime mass and volume based on the smallest diameter, as best shown in Eq. (6) and Eq. (7), as shown in Fig. 1 and Fig.2:

$$M = 0.0008 c^{2.4925} \quad R^2 = 0.82 \quad (6)$$

$$V = 0.0011 c^{2.8091} \quad R^2 = 0.87 \quad (7)$$

The mass and volume model recommended for sizing of Limes based on any one projected area, as in Eq. (8) and Eq. (9), is polynomial and linear form, respectively:

$$M = -1E-07 Pc^2 + 0.0048 Pc + k_3, \quad R^2 = 0.83 \quad (8)$$

$$V = 0.0218PC - 5.2006, \quad R^2 = 0.90 \quad (9)$$

Acknowledgement

The authors would like to express their gratitude and their sincere appreciation to the Deputy of Agronomy Departments for their cooperation and laboratory support at the Razi University of Kermanshah.

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(Published in September 2012)