Modeling the soil cutting process in rotary tillers using finite element method

N. Alavi\(^*\) and R. Hojati

Department of Agricultural Machinery, Shahid Bahonar University of Kerman, Kerman, Iran


Using rotary tillers as one of efficient tillage machinery in gardens is remarkable. Aggregate powdering due to applying additional stresses to the soil is considered undesirable when using rotary tillers. In this study, the process of soil cutting is modeled using Finite Element Method (FEM) considering the effect of forward speed, rotary speed and soil moisture content on rate of stress applied to the soil. It was concluded that increasing forward speed led to decreasing the applied stress and increasing the rotary speed and soil moisture content led to increasing the applied stress on soil. The rate of stress in different conditions was compared with the allowed stress in soil and also the additional stress that led to aggregate powdering was computed.

**Key words:** Roitary tiller, Finite element method, Forward speed, Rotary speed, Moisture content

**Introduction**

Tillage, a process of applying energy to the soil to change soil physical condition or to disturb soil for some other purpose, is one component in any system of soil management for crop production. Tillage processes are used in crop production for several purposes, such as loosening soil to create a seedbed or a root bed, moving soil to change the micro topography, or mixing soil to incorporate amendments. Soil tillage has always been a major research area in agriculture. As tillage operation is a procedure for breaking up soil, soil failure depends mainly upon the soil properties, tool geometry, and cutting speed. Almost all of the soil cutting tools used in agriculture have been developed by field experiment and by trial and error. Experimental and theoretical analysis techniques are essential to develop efficient tillage or soil cutting tools which will require less energy and still provide a satisfactory soil condition for crop emergence and growth. The field experiment allows prototype verification with specialize instrumented equipment growing in popularity to accelerate the

\(^*\) Corresponding author: Seyed Naser Alavi; e-mail: seyed.naser.alavi@gmail.com
production process requirements and the desire to decrease prototype construction and verification. Analytical and empirical models are still used to solve soil–tool interaction applications, but many of these models are 2D and are theoretically suitable only for very wide tools (Mouazen and Nemenyi, 1998). A few 3D models to predict narrow tillage tool behavior in soils are available (Abu-Hamdeh and Reeder, 2003, Abo-Elnor et al., 2003 and 2004).

The majority of these models, however, are for slow-moving tools and do not take into consideration the speed effects. Most tillage operations, on the other hand, are performed at speeds in the range 4–10 km/h, where the soil forces on the tillage tools are expected to vary with tool speed. The latest advances in computer performance are proving to be promising for numerical approaches to tool design such as the finite element method. Much work has been reported on the static analysis of tillage problems using the finite element method (Chi and Kushwaha, 1990; Fielke, 1999; Mouazen and Nemenyi, 1999; Abu-hamed and Reeder, 2003; Abo-Elnor et al., 2004). This method was shown to be capable of simulating dynamic effect of forward speed and rotary speed and soil moisture content on stress applied to the soil through the cutting process in rotary tillers.

Materials and methods

The finite element model have the capability to predict the effect of forward speed, rotary speed, and soil moisture content on stresses producing duration cutting process in rotary tillers. The details of these analyses are included in the following sections.

Constitutive relationships

The hyperbolic model developed by Duncan and Chang (1970) to represent a typical stress–strain relationship was used in this study. The model represents the nonlinear elastic behavior of soil and the tangent modulus of elasticity in this model is expressed as a function of soil stress level and soil strength. This model was selected for its generality as well as for the convenience involved in the determination of the model parameters using triaxial tests. The hyperbolic model is given by:

\[
E_t = KP \left[ \frac{\sigma_1}{P_a} \right] \left[ 1 - \frac{R_f (\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f} \right]^2
\]

where \( E_t \) is the tangent modulus of elasticity, \( P_a \) the atmospheric pressure, \( \sigma_1 \) the major principal stress in soil, \( \sigma_3 \) the minor principal stress in soil,
\[ (\sigma_1 - \sigma_3)_{f} = (\sigma_1 - \sigma_3) \] at soil failure, \( R_f \) the failure ratio defined as the ratio of ultimate deviatoric stress to the soil strength, and \( K, n \) the dimensionless numbers determined from triaxial test results. In Duncan’s equation, the tangent modulus, \( E_t \) of soil was expressed as a function of the major and minor principle stresses. Established a relation between soil strain rate and shear stress by means of statistical mechanics as follows:

\[
\ln \dot{\varepsilon} = \alpha + \beta \left[ \frac{1}{2} (\sigma_1 - \sigma_3) \right] \tag{2}
\]

where \( \dot{\varepsilon} \) is the actual strain rate in soil and \( \alpha, \beta \) are the equation coefficients. The following expression for a modified tangent modulus based on Eqs. (1) and (2):

\[
E_t = K_P \left[ \frac{\sigma_3}{P_0} \right] \left[ 1 - \frac{R_f (\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_{f_0}} \right] \left( 1 + B \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right) \tag{3}
\]

Where \( \dot{\varepsilon}_0 \) is the maximum strain rate at conventional triaxial apparatus, \( (\sigma_1 - \sigma_3)_{f_0} = (\sigma_1 - \sigma_3) \) at soil failure from conventional triaxial apparatus and \( B \), the coefficient relating to the strain rate effect.

The forces due to the motion of the blade affected by adhesion and friction between soil and the surface of the tillage tool.

The shear stress at soil failure is given by:

\[
\tau_f = C_a + \sigma_n \tan \theta \tag{4}
\]

where \( C_a \) is the adhesion between soil and cutting tool, \( \sigma_n \) the normal stress, and \( \theta \) the friction angle between the soil and tool.

Duncan and Chang (1970) stated that two material properties, namely the tangent modulus and Poisson’s ratio are necessary to completely describe the mechanical behavior of any material under a general system of changing stresses. The following equation proposed by Chi and Kushwaha, (1991) calculates values of Poisson’s ratio, \( \nu \), as a linear function of stress level and soil strength:

\[
\nu = a + b \left[ \frac{(1 - \sin \theta)(\sigma_1 - \sigma_3)}{2C \cos \theta + 2\sigma_3 \sin \theta} \right] \tag{5}
\]
Boundary condition applied

The soil-cutting model considered in the analysis was idealized with triangular constant strain elements because of their simplicity and convenience for nonlinear material. A program written in MATLAB was developed to present the piece of soil cutting with the blade. Movement of the blades in rotary tillers is the combination of linear movement and rotary movement. Geometry trajectory of this movement is presented using equation (6).

\[
\begin{align*}
  x(t) &= v_t t + R \cos(\omega t) \\
  y(t) &= R \sin(\omega t)
\end{align*}
\]

(6)

Where \( v_t \) is forward speed, \( R \) the cutting blade (blade length adding radius of rotor), \( \omega \) angular velocity of the rotor and \( t \) time. Angular velocity of the rotor can be calculated using equation (7).

\[
\omega = \frac{2\pi n}{60}
\]

(7)

Where \( n \) is revolution per minute of the rotor. The fixed assumed parameters were the cutting length, operating depth, number of blades on flange and type of the soil. The L-shape blade was considered as the blade used in this study. Type of the soil was sandy-silt. The variable parameters were forward speed, rotary speed and soil moisture content. The rates of the parameters were:

Cutting length (22 cm)
Operating depth (16 cm)
Number of blades on flange (6 blades)
Forward speed (2.23 km/hr and 3.41 km/hr)
Rotary speed (183 rpm and 251rpm)
Soil moisture content (10-12%, 12-14% and 14-16%)

The shapes of piece of soil cutting with the blades for different condition are presented in Figs.1 to 4.

Figures 5 and 6 illustrate the finite element mesh for the piece of soil cutting with the blade of rotary tiller in forward speed of 2.23 km/hr and rotary speed of 183 rpm. The region of influenced in the analysis had a length in the longitudinal direction of seven the times the tool operating depth and a depth of three times the tool operating depth.

Finite element formulation

The general matrix differential equations for time dependent problems can be expressed as follows (Cook et al., 1988):
\[ M \ddot{a} + C \dot{a} + Ka + f = 0 \]  \hspace{1cm} (8)

where \( f \) is the external load vector, \( \dot{a}, \dot{a}, a \) are the nodal acceleration, velocity and displacement vectors, respectively, and \( M, C, K \) the mass, damping and stiffness matrix, respectively.

For tillage problems, the soil forces are determined by the \( f \) vector which can be considered to comprise of three components: acceleration, damping and static equilibrium.

Since soil is nonlinear elastic material, the incremental method was implemented into finite element program to solve the nonlinear behavior of soil and interaction between the soil particles.

It was assumed that only shear and tensile stresses cause failure in agricultural soils. Shear failure occurred at one Gauss point when the difference between the major and minor principle stresses at this point exceeded the maximum shear strength, and the tangent modulus was modified to a small value (\( 10^{-6} \) times the initial modulus).

The force applied with blade to the soil was calculated using the needed draft for rotary tiller in different net speed (summation of forward speed and linear speed of blades). The equations are presented as follows:

\[ D = c_1 + c_2 s^2 \]  \hspace{1cm} (9)

where \( D \) is the specific draft of soil and \( c_1, c_2 \) the constants related to the type of soil. A program written in MATLAB was presented to calculate the force applied to the soil. Six points were selected in the soil pieces which have the same geometry in different condition of forward speed and rotary speed. The coordinates of six points are presented in Table 1.

Table 1. Coordinates of selected points

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Label</th>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>X[cm]</td>
<td>Y[cm]</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-12</td>
<td>A</td>
</tr>
<tr>
<td>18</td>
<td>-14</td>
<td>B</td>
</tr>
<tr>
<td>16</td>
<td>-16</td>
<td>C</td>
</tr>
<tr>
<td>13</td>
<td>-18</td>
<td>D</td>
</tr>
<tr>
<td>9</td>
<td>-20</td>
<td>E</td>
</tr>
<tr>
<td>5</td>
<td>-21</td>
<td>F</td>
</tr>
</tbody>
</table>

PLAXIS software was used to model the soil behavior while using rotary tillers. Four models were prepared to describe the conditions of different
forward speed and rotary speed. Soil parameters were measured in laboratory. The obtained rates are presented in Table 2.

Table 2. Soil parameters

<table>
<thead>
<tr>
<th>Moisture content</th>
<th>$\rho_d$ [kg/m(^3)]</th>
<th>$\rho$ [kg/m(^3)]</th>
<th>$\phi$ [deg]</th>
<th>$C$ [N/m(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-12</td>
<td>1610</td>
<td>1770</td>
<td>22</td>
<td>9000</td>
</tr>
<tr>
<td>12-14</td>
<td>1610</td>
<td>1800</td>
<td>21.46</td>
<td>9300</td>
</tr>
<tr>
<td>14-16</td>
<td>1610</td>
<td>1833</td>
<td>20.61</td>
<td>9800</td>
</tr>
</tbody>
</table>

Results and discussion

Results of the finite element analysis included the calculation of the stress in each selected point. The results obtained of modeling are presented in Tables 3 to 6. Increasing forward speed led to decrease the applied stress to the selected points. For example, stress in point 1 with label A was -15560 pa while forward speed 2.23 km/hr, rotary speed 183 rpm and moisture content 10-12% and it became -15031 pa while forward speed 3.41 km/hr, rotary speed 251 rpm and moisture content 10-12% . It is related to increase the piece of soil area while decreasing forward speed. The piece of soil area in forward speed 2.23 km/hr and rotary speed 183 rpm is 107.94 cm\(^2\) while it is 163.14 cm\(^2\) in forward speed 3.41 km/hr and rotary speed 183 rpm. The area of the piece of soil in different conditions is presented in Table 3. Jiang et al. (2010) also reported the same results. They showed that the appropriate unit to speed up forward speed, will be conducive to the improvement of energy efficiency.

Table 3. Piece of soil Area

<table>
<thead>
<tr>
<th>Observation</th>
<th>Forward speed [km/hr]</th>
<th>Rotary speed [rpm]</th>
<th>Area [cm(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.23</td>
<td>183</td>
<td>107.94</td>
</tr>
<tr>
<td>2</td>
<td>2.23</td>
<td>251</td>
<td>78.45</td>
</tr>
<tr>
<td>3</td>
<td>3.41</td>
<td>183</td>
<td>163.14</td>
</tr>
<tr>
<td>4</td>
<td>3.41</td>
<td>251</td>
<td>121.49</td>
</tr>
</tbody>
</table>

Increasing soil moisture content led to increase the applied stress to the selected points. For example, stress in point 5 with label E was -22214 pa while forward speed 3.41 km/hr, rotary speed 251 and moisture content 10-12%, while it became -22788 pa when moisture content was 12-14% and -23766 pa when moisture content was 14-16%. It is related to increase the soil weight due to increase of soil moisture content. Rate of stress in selected points in different
condition are presented in Tables 4 to 7. The graphs of stress applied in selected points are shown in Figs. 7 to 10. Shmulevich et al. (2007) model predicted the same behavior. They showed soil moisture had a significant effect on forces and stresses on soil cutting blades.

Table 4. Stress in selected points in forward speed 2.23 km/hr and rotary speed 183 rpm

<table>
<thead>
<tr>
<th>Point</th>
<th>Label</th>
<th>Coordinate</th>
<th>Moisture Content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X[cm] Y[cm]</td>
<td>10-12%</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>20 -12</td>
<td>-15560</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>18 -14</td>
<td>-16161</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>16 -16</td>
<td>-16733</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>13 -18</td>
<td>-22166</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>9  -20</td>
<td>-22476</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>5  -21</td>
<td>-23352</td>
</tr>
</tbody>
</table>

Table 5. Stress in selected points in forward speed 2.23 km/hr and rotary speed 251 rpm

<table>
<thead>
<tr>
<th>Point</th>
<th>Label</th>
<th>Coordinate</th>
<th>Moisture Content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X[cm] Y[cm]</td>
<td>10-12%</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>20 -12</td>
<td>-15748</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>18 -14</td>
<td>-16236</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>16 -16</td>
<td>-16950</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>13 -18</td>
<td>-22218</td>
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<tr>
<td>5</td>
<td>E</td>
<td>9  -20</td>
<td>-23714</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>5  -21</td>
<td>-24275</td>
</tr>
</tbody>
</table>

Table 6. Stress in selected points in forward speed 3.41 km/hr and rotary speed 183 rpm

<table>
<thead>
<tr>
<th>Point</th>
<th>Label</th>
<th>Coordinate</th>
<th>Moisture Content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X[cm] Y[cm]</td>
<td>10-12%</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>20 -12</td>
<td>-15031</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>18 -14</td>
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<td>4</td>
<td>D</td>
<td>13 -18</td>
<td>-21475</td>
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<tr>
<td>5</td>
<td>E</td>
<td>9  -20</td>
<td>-22063</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>5  -21</td>
<td>-22659</td>
</tr>
</tbody>
</table>
### Table 7. Stress in selected points in forward speed 3.41 km/hr and rotary speed 251 rpm

<table>
<thead>
<tr>
<th>Point</th>
<th>Label</th>
<th>Coordinate</th>
<th>Moisture Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 A</td>
<td>20</td>
<td>-12</td>
<td>-15095</td>
</tr>
<tr>
<td>2 B</td>
<td>18</td>
<td>-14</td>
<td>-15413</td>
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<td>3 C</td>
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<td>-16228</td>
</tr>
<tr>
<td>4 D</td>
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<td>-18</td>
<td>-21679</td>
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<tr>
<td>5 E</td>
<td>9</td>
<td>-20</td>
<td>-22214</td>
</tr>
<tr>
<td>6 F</td>
<td>5</td>
<td>-21</td>
<td>-23294</td>
</tr>
</tbody>
</table>

Fig. 1. Piece of soil in forward speed 2.23 km/hr and rotary speed 183 rpm.

Fig. 2. Piece of soil in forward speed 2.23 km/hr and rotary speed 251 rpm.

Fig. 3. Piece of soil in forward speed 3.41 km/hr and rotary speed 183 rpm.

Fig. 4. Piece of soil in forward speed 3.41 km/hr and rotary speed 251 rpm.
Fig. 5. Finite element mesh of soil in forward speed 2.23 km/hr and rotary speed 183 rpm.

Fig. 6. Selected points in forward speed 2.23 km/hr and rotary speed 183 rpm.

Fig. 7. Graph of stress in selected points in forward speed 2.23 km/hr and rotary speed 183 rpm.

Fig. 8. Graph of stress in selected points in forward speed 2.23 km/hr and rotary speed 251 rpm.
Increasing rotary speed led to increase the applied stress to the selected points. For example, stress in point 2 with label B was -16536 pa while forward speed 2.23 km/hr, rotary speed 183 rpm and moisture content 12-14% and it became -16635 pa while forward speed 2.23 km/hr, rotary speed 251 rpm and moisture content 12-14%. It is related to increase the load forced with blade to the soil and decrease the piece of soil while increasing the rotary speed. The following studies also predicted the same behavior. Abu-Hamdeh and Reeder (2003) showed that increasing plowing speed increased the draft and side forces for a disk plow. They showed that the amount of increase was affected by soil type. Abo-Elnor et al. (2004) used three-dimensional dynamic finite element analyses to simulate soil–tool interaction and study the effect of cutting speed and cutting acceleration on predicted cutting forces. A series of models were analyzed with various cutting speeds and cutting accelerations using three-dimensional models. Results showed the significant effect of cutting acceleration on cutting forces. Jiang et al. (2010) also reported the same results. They showed when tool rotation speed increased, the average distortion increased. Singh and Sharda (2004), also showed the same results.

Conclusions

Modeling soil–tillage interaction is a complex process due to the following: the nonlinearity of soil material; the spatial variability of the soil media; and the contact phenomenon and flow that occur at the interface zone between the soil and the tillage tool. The effect of forward speed, rotary speed and soil moisture content on stress applied to the soil was investigated using finite element method. Based on the results from this study, the following conclusions can be drawn as follows:- increasing forward speed decreased the
stress applied to the soil, increasing rotary speed of the rotor increased the stress applied to the soil and increasing the soil moisture content increased the stress applied to the soil.

References


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